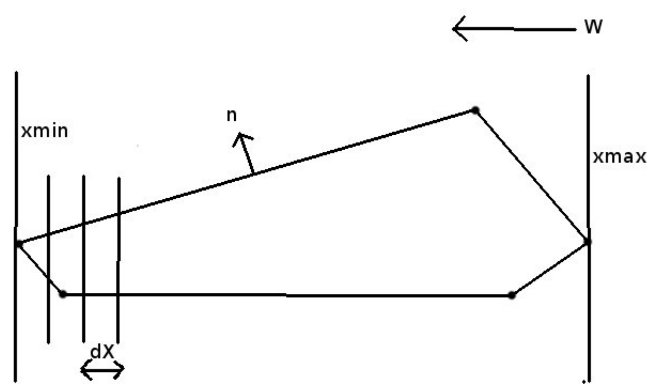
1 Lift and Drag derivation for any convex hull

The end goal of this short mathematical thought experiment is to determine a fast solution to the parallel and perpendicular forces on a wing as a result of wind. These forces are known as drag and lift respectively. The suggested method involves the use of convex hulls to describe wing geometry, and a finite discrete integration method using Bernoulli's principle to determine net forces. Although rotational forces will not be covered in this derivation, individual components of drag and lift could be used in conjunction with rotational inertia of a wing shape to determine torque and rotational acceleration. This would make it possible to produce an accurate model of motion in a windy envorinment for any two dimensional convex shape.

1.1 Geometry

The following figure shows the general geometry for such a convex hull:



1.2 Lift

Lift is the force perpendicular to the wind direction.

From Bernoulli's Equation:

$$\frac{pv^2}{2} + \rho gh + \frac{p}{\rho} = const \tag{1}$$

where ρ is the pressure. In this case (high altitude) we can take the ρgh term to be constant giving:

$$\frac{pv^2}{2} + \frac{p}{\rho} = const \tag{2}$$

which can be factorised in p to give:

$$p(\frac{v^2}{2} + \frac{1}{\rho}) = const \tag{3}$$

Giving p as:

$$p = \frac{const}{\frac{v^2}{2} + \frac{1}{\rho}} \tag{4}$$

The velocity of the wind across a surface segment can be given as the inverse of the dot product between the surface normal multiplied by the normal direction of the wind or $|w|(\hat{n}.\hat{w})^{-1}$, which can be substituted into 4 to give:

$$p = \frac{const}{\frac{|w|^2}{2.(\hat{n}.\hat{w})^2} + \frac{1}{\rho}} \tag{5}$$

This gives a pressure value at a given point on a surface based on its normal. At this stage a pressure based approach can be used to determine the total lift on the wing by discretely integrating:

$$L = \oint p\hat{n}.\hat{w}dA \tag{6}$$

Which gives:

$$L = \Sigma_{x_{min}}^{x_{max}} \frac{\hat{n}.\hat{w}.C}{\frac{|w|^2}{2.(\hat{n}.\hat{w})^2} + \frac{1}{\rho}} dX \tag{7}$$

Where C is an arbitrary constant that will affect the strength of the force.

By stepping through each step dX across the shape from x_{min} to x_{max} the total lift force on any object can be determined, given any wind direction.

1.3 Drag

The drag is the force parallel to the direction of the wind.

To simplify the problem, only faces with normals facing into the wind $\hat{n}.\hat{w} < 0$ will be considered. Instead of stepping across from x_{min} to x_{max}, we will step from y_{min} to y_{max}.

Drag is given by:

$$D = -\frac{1}{2}\rho A C_d (v.v)\hat{v} \tag{8}$$

where A is the area relative to the wind direction. We can calculate A to be $C_d dY(\hat{n}.\hat{w})$ where C_d is some scalar. Using this we get:

$$D = -\Sigma_{y_{min}}^{y_{max}} C_d |w - v|^2 \hat{n}.\hat{w}\hat{v}dY \tag{9}$$

Where v is the velicity of the wing. In other words, to calculate the net drag we simply step over in the y direction and determine the drag force at each step.

In two dimensions this is equivalent to using dX, and as such because wind direction should be arbitrary, dY and dX are interchangable as a unit of length perpendicular to the wind. Upon intergration the outcome for both drag and lift will become equivalent. For this reason it would be far easier to wholly use vector notation!

It should be noted that this is not an accurate simulation, but many of the arguments are valid and thus this method may produce realistic effects when implemented. The processing time for such a problem is linear in N, where N is the number of subdivisions and as such is a fast solution to drag and lift.